Component Mode Synthesis of a Vehicle Structural-Acoustic System Model

Shung H. Sung* and Donald J. Nefske*
General Motors Research Laboratories, Warren, Michigan

This paper describes the application of the component mode synthesis technique to develop an analytical structural-acoustic system model of the automotive vehicle. The system model combines an acoustic finite element model of the automobile passenger compartment cavity with finite element and modal models of the vehicle structural system. The model can be solved for frequency, random, and transient response to predict the low-frequency interior noise that occurs during actual operating conditions of the vehicle. The theoretical formulation of the model is described, as well as an experimental verification for random road input. The paper presents analytical capabilities for calculating the modal and substructure participation factors to the interior noise response and illustrates the use of the participation factors in identifying modifications for interior noise reduction. Finally, the application of the model in a computer-aided procedure for the structural-acoustic design of the vehicle is described.

Introduction

FOR large-scale structures, it is common practice to subdivide the structure into several smaller components or substructures for the purpose of analysis. The modal characteristics of the individual substructures can be obtained analytically, experimentally, or by applying numerical techniques such as the finite element method. Then, by employing the modal synthesis technique, the substructure modal characteristics are combined into one system to predict the overall structural dynamic response.[1-8]

Similarly, the modal synthesis technique has been developed for coupled structural-acoustic systems.[9-14] While the structural system is usually described in terms of displacement variables, the acoustic system is most often described in terms of pressure (or equivalently, force) variables. When the two systems are coupled, appropriate dynamic boundary conditions have to be satisfied at the interfacial region.[15] Recently, a computational procedure has been developed to calculate the structural-acoustic coupling for any complicated finite element models of the structural and acoustic systems.[16] By incorporating this coupling in a modal synthesis formulation, a coupled structural-acoustic modal model has been developed. This capability was applied to form a modal model of an automotive vehicle body and its enclosed passenger compartment cavity to predict the interior noise response for single, harmonic, forcing inputs applied to the body structure.[16,17]

As an extension of the present capabilities, this paper describes the development of a coupled structural-acoustic system model of the complete vehicle. This model includes, in addition to the vehicle body and passenger compartment cavity, a model of the chassis system consisting of the frame, powertrain, suspension, and various other substructures that comprise the vehicle. In the formulation, the component mode synthesis method is first applied to combine the dynamic characteristics of the vehicle chassis system with the vehicle body model to formulate a modal model of the vehicle structural system. The structural system model has been previously used to evaluate the vehicle system modes, as well as to investigate the vehicle dynamic response.[18] To couple the passenger compartment acoustic model with the structural system model, this paper derives the mathematical transformation to generate the corresponding coupling between the structural system modes and the cavity modes. With this new coupling, the modal synthesis procedure is used to form a coupled modal model of the vehicle structural-acoustic system. Finally, an analysis procedure is developed to identify the participation of the component modes and substructures to the structural-acoustic response for the vehicle subject to either frequency or random loading inputs.

As an example application, a structural-acoustic system model of a representative automotive vehicle is developed. The model provides a means of predicting the low-frequency interior noise response in the passenger compartment for inputs occurring during actual operating conditions of the vehicle. As an experimental evaluation of the model, the predicted interior noise is compared with the measured noise in an actual vehicle being driven on the road. The participation analysis is then applied to the vehicle model to identify the noise sources and paths that should be modified for interior noise reduction. As an example, structural modifications that reduce the participation of the dominant vehicle mode and body panels are evaluated. To conclude, the application of the model in a computer-aided procedure for the structural-acoustic design of the automotive vehicle is illustrated.

Theoretical Development

Structural System

For an automotive structure, the vehicle components are typically modeled using either the finite element or modal method. By combining both the finite element and modal models, the equations of motion for the vehicle structural system can be expressed as

$$[M]\{\ddot{w}\}+[C]\{\dot{w}\}+[K]\{w\}=\{F\} \tag{1}$$

where $\{w\}$ represents the displacement response which consists of both physical and modal degrees of freedom. The matrices $[M]$, $[C]$, and $[K]$ represent the inertia, damping, and stiffness matrices for the component modes and substructures, respectively, and $\{F\}$ is the vector of external forces applied to the vehicle. In the formulation of Eq. (1) the interfacial constraint forces have been excluded from the

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*Staff Research Engineer, Engineering Mechanics Department.

external forcing vector $\{F\}$. Instead, the displacement constraints for interconnected substructures, as well as the modal representation of certain physical degrees of freedom, are taken as linear constraint equations of the form

$$[E]\{w\} = \{0\} \tag{2}$$

where $[E]$ includes the multiple constraint conditions between the physical and modal degrees of freedom. The structural system modes are obtained by solving Eq. (1) for the undamped, free vibration (i.e., $[C]=0$, $\{F\}=0$), while subjected to the constraint conditions in Eq. (2). These modes are represented in matrix form as

$$[W] = [\{w_1\}\{w_2\}\ldots\{w_i\}\ldots\{w_I\}] \tag{3}$$

where I is the total number of system modes.

The structural system response can now be expressed in terms of the system modes through the transformation

$$\{w\} = [W]\{\zeta\} \tag{4}$$

where $\{\zeta\}$ is a vector of the system modal degrees of freedom that determine the relative participation of the various system modes in producing the response. By substituting Eq. (4) into Eq. (1) and premultiplying Eq. (1) by $[W]^T$ (i.e., the transpose of $[W]$), one obtains the modal form of the equations of motion for the structural system

$$[m]\{\ddot{\zeta}\} + [c]\{\dot{\zeta}\} + [k]\{\zeta\} = \{f\} \tag{5}$$

where $[m]$, $[c]$, and $[k]$ represent the structural-system modal mass, damping, and stiffness matrices, respectively, and $\{f\} = [W]^T\{F\}$ the system modal forcing vector. Upon solving Eq. (5) for $\{\zeta\}$, the modal and physical response of any component of interest can be obtained through Eq. (4) as

$$\{\eta\} = [\beta]\{\zeta\} \tag{6}$$

$$\{u\} = [\phi]\{\eta\} \tag{7}$$

where $\{\eta\}$ is the modal response of a particular component and $\{u\}$ its physical displacement. Here, $[\beta]$ represents the matrix of system modes associated with the particular component's modal degrees of freedom (i.e., $[\beta]$ is a particular partition of $[W]$) and $[\phi]$ represents the component modes with each column corresponding to the displacement distribution of a particular mode.

Coupled Structural-Acoustic System

In the above formulation, the modal response vector $\{\eta\}$ determines the physical displacement $\{u\}$ of any component of interest. If this particular component is the vehicle body structure, its vibration will excite a pressure response in the interior passenger compartment.[17] The governing equation for determining this pressure response from an acoustic finite element model of the interior passenger compartment is[16]

$$[Q]\{\ddot{p}\} + [D]\{\dot{p}\} + [H]\{p\} = -[A]\{\ddot{u}\} + \{F_2\} \tag{8}$$

where $\{p\}$ is the vector of sound pressures at the grid points of a finite element mesh that discretizes the air volume and $[Q]$, $[D]$, and $[H]$ the acoustic inertia, damping, and stiffness matrices of the air, respectively. The right-hand side of this equation represents forces that generate sound pressure in the air volume; these include the body panel accelerations $\{\ddot{u}\}$, which are transformed to acoustic excitations through the interface surface area matrix $[A]$, and also the force vector $\{F_2\}$, which represents interior acoustic sources, such as loudspeaker excitations.

Equation (8) can be reduced to modal form by the transformation

$$\{p\} = [\psi]\{\xi\} \tag{9}$$

where $[\psi]$ are the rigid-wall acoustic modes and $\{\xi\}$ the acoustic modal degrees of freedom. By substituting Eqs. (7) and (9) into Eq. (8) and premultiplying the resulting equation by $[\psi]^T$ (i.e., the transpose of $[\psi]$), one obtains the modal form of the acoustic equations of motion for the passenger compartment cavity,

$$[q]\{\ddot{\xi}\} + [d]\{\dot{\xi}\} + [h]\{\xi\} = -[a]\{\ddot{\eta}\} + \{f_2\} \tag{10}$$

where $[q]$, $[d]$, and $[h]$ represent the acoustic modal mass, damping, and stiffness matrices, respectively, $[a] = [\psi]^T[A][\phi]$ is the structural-acoustic coupling between the vehicle body modes and the acoustic modes, and $\{f_2\} = [\psi]^T\{F_2\}$ is the modal forcing vector associated with acoustic sources.

To express the structural-acoustic coupling in Eq. (10) in terms of the system modal degrees of freedom $\{\zeta\}$, one may substitute Eq. (6) into Eq. (10). The acoustic modal equations of motion then become

$$[q]\{\ddot{\xi}\} + [d]\{\dot{\xi}\} + [h]\{\xi\} = -[t]\{\ddot{\zeta}\} + \{f_2\} \tag{11}$$

where $[t] = [a][\beta]$ represents the modal coupling matrix which transforms the vehicle system response to acoustic excitations of the interior passenger compartment. Because, in the structural system model, the vehicle body response is coupled to the interior pressure loading, Eq. (5) is also rewritten to explicitly represent these pressure loading effects:

$$[m]\{\ddot{\zeta}\} + [c]\{\dot{\zeta}\} + [k]\{\zeta\} = [t]^T\{\xi\} + \{f_1\} \tag{12}$$

where $\{\xi\}$ are the acoustic modal degrees of freedom and $\{f_1\}$ a modal vector associated with only the external forces applied to the vehicle. By solving Eqs. (11) and (12) simultaneously, one can obtain the coupled structural and acoustic vehicle modal response. The sound pressure response in the passenger compartment is then obtained by evaluating Eq. (9) for interior points of interest.

Participation Factors

In the automotive vehicle, the variety of loading inputs and the complexity of the vibration transmission paths complicate the structural-dynamic behavior that produces the interior noise response. However, the structural-acoustic system model provides a means of identifying the transmission paths and the contribution the vehicle modes and substructures make to the resultant noise. In the following, the mathematical formulation for obtaining the vehicle modal and body panel participation factors will be derived. In the next section, these will be utilized to identify various modifications that alter the mode or panel response to reduce the interior noise.

As shown in Ref. 16, for harmonic forcing inputs, the acoustic response $\{p\}$ in Eq. (9) can be partitioned in the data recovery phase of the computations in various ways, such as

$$\{p\} = \sum_L \{p_l\}; \quad \{p\} = \sum_M \{p_m\}; \quad \{p\} = \sum_N \{p_n\} \tag{13}$$

where L is the total number of acoustic modes, M the total number of vehicle body modes, and N the total number of panels into which the cavity surface is divided. Equation (13) is applied to a particular loading case. For random response, there exist correlated loading inputs (both in time and location) from the tires, engine, etc., and the acoustic pressure response is expressed in terms of its power spectral density (PSD) given by[19]

$$\{P(\omega)\} = \sum_{r=1}^{L_r} \sum_{s=1}^{L_s} \{(p_r)^* Q_{rs}(\omega)(p_s)\} \tag{14}$$

where p_r and p_s represent the frequency response of the sound pressure for harmonic loading cases r and s, respectively, $Q_{rs}(\omega)$ the spectral density of the forcing input for the loading cases, L_r and L_s the total number of loading cases, and $(\;)^*$ the complex conjugate. For the vehicle subject to road input only, Q_{rs} will represent the tire inputs from the right and left side of the vehicle.[20]

For each loading case, p_r and p_s are determined from Eq. (13), so that by substituting from this equation one can express Eq. (14) as

$$\{P(\omega)\} = \left\{ \sum_{i=1}^{I} P_i(\omega) \right\} \tag{15}$$

where P_i is the contribution to the sound pressure PSD from acoustic mode l, vehicle body mode m, or body panel n. Accordingly, I then represents the total number of acoustic modes L, vehicle body modes M, or vehicle body panels N. Since P_i in Eq. (15) is complex, it is convenient to define a normalized participation factor R_i as

$$R_i(\omega) = [P_i(\omega) \cdot P^*(\omega)] / |P(\omega)|^2 \tag{16}$$

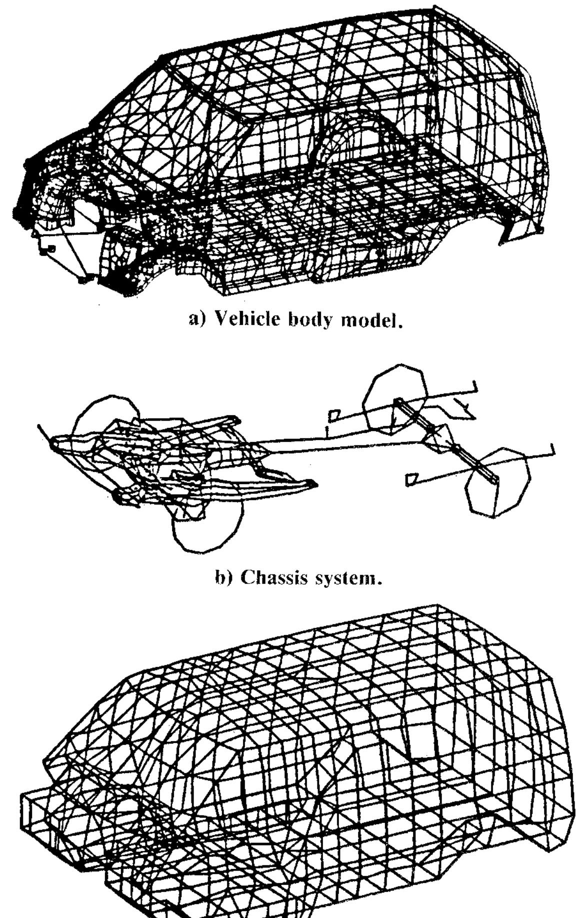
which is used to rank the participation of $P_i(\omega)$ relative to a referenced noise response $P(\omega)$. In this equation, the sign of $R_i(\omega)$ has significant physical meaning, where a positive value indicates sound energy is produced by the particular mode or panel and a negative value indicates sound energy is reduced due to the excitation of this particular mode or panel. Equation (16) also applies in the case of frequency response, which corresponds to random response with a single forcing input, and can be used to rank the participation of the modes or panels.

Application to Automotive Vehicle

Structural-Acoustic System Model

As an illustrative application of the methodology, a structural-acoustic system model of an automotive van-type vehicle was developed and experimentally evaluated. Figure 1 shows the structural finite element model of the vehicle body, the geometrical outline of a model of the chassis system, and the discretization of the passenger compartment surface from a three-dimensional acoustic finite element model of the compartment cavity. The vehicle body model represents the primary, untrimmed structure (the so-called "body-in-white"), plus exterior sheet metal, doors, windows, seats, and other trim and accessories. The chassis model represents the frame, suspension, powertrain, and various components that form the mechanical parts and undercarriage of the vehicle. The body and chassis systems, as well as their components, are connected at discrete mount and joint locations and by local compliances at frame and sheet metal connections. For components such as the vehicle body, frame, and suspension, finite element models were developed and used to evaluate the static and dynamic performance of these components. However, for vehicle components possessing highly complex structural or material characteristics, such as the powertrain, experimental modal models were necessary to accurately represent the substructure behavior. The acoustic finite element model of the passenger compartment represented the van's interior geometry and included the particular seat configuration.

To develop the structural-acoustic system model of the vehicle shown in Fig. 1, the synthesis procedure indicated schematically in Fig. 2 was employed. This procedure involved synthesizing the structural system modes, the passenger compartment acoustic modes, and the coupling between these two sets of modes. The structural system modes were calculated by synthesizing the vehicle body modes with the finite element and modal models of the chassis system. Then, the coupling between the structural system modes and the cavity modes was generated by performing the appropriate mathematical transformations. With the models in



a) Vehicle body model.

b) Chassis system.

c) Acoustic model.

Fig. 1 Vehicle structural-acoustic system model.

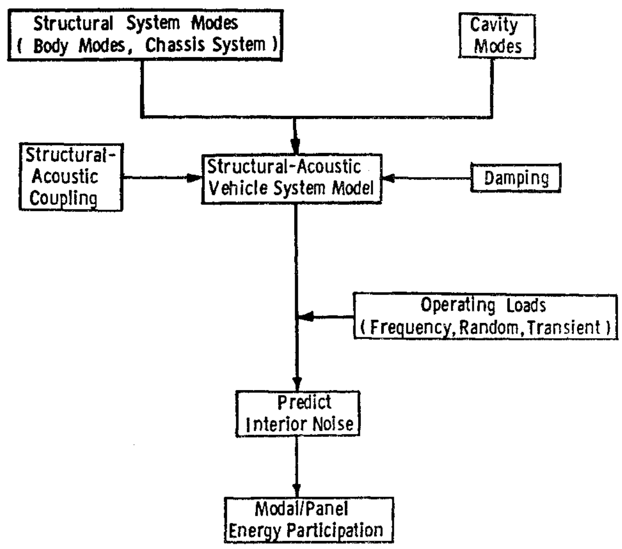


Fig. 2 Synthesis and analysis procedures for structural-acoustic system model.

Fig. 1, some cavity surface regions were not represented in the structural body model and the corresponding structural-acoustic coupling effects were neglected. For instance, in a previous study,[21] detailed structural finite element models were not included for the doors, seats, instrument panel, and engine cover, and only 70% of the cavity surface area was coupled with the surrounding structure. In this paper, door models were included in the body structural model, resulting in a model with 90% of the cavity surface area coupled. To complete the model, damping of the vehicle body modes and compartment acoutic modes is also required and was included as modal damping. The acoustic modal damping was determined experimentally from the measured loudspeaker response in a similar passenger compartment,[23] while the structural modal damping was inferred from the measured vibration response of a similar vehicle body structure subjected to shaker excitation.[16] Finally, the synthesized model was solved for the structural and acoustic response for frequency, random, and transient inputs, using the standard capabilities of the MSC/NASTRAN computer code.[22] In the postprocessing phase, the participation analysis procedures were employed to calculate the modal and panel contribution to the noise response at various interior points.

The synthesized structural-acoustic system model for the vehicle was then used to simulate the road-induced interior noise response. The number of structural and acoustic modes included in the synthesized system model is shown in Fig. 3, where 221 structural system modes and 8 passenger compartment acoustic modes were utilized. This resulted in a convergent solution of the system model within the frequency range below 100 Hz. As an experimental evaluation, comparison was made between the predicted sound pressure level at the front passenger's ear location vs the measured response for an actual vehicle traveling at 35 mph over a randomly rough, spalled concrete road. As Fig. 4 shows, the model predicts the important acoustic peak responses (e.g., those at 34 and 54 Hz) that are also observed in the measured data, as well as the overall trend of the acoustic behavior. In particular, the overall noise level predicted by the system model for the 20-80 Hz frequency range is within 0.3 dB of the overall noise level measured in the road test. Finally, by comparing the predicted results in Ref. 21 (without doors coupled) with those in Fig. 4, it is observed that the accuracy of the model is significantly improved when the structural vibration inputs from the doors are included.

Modal and Substructure Participation

The modal and substructure participation analysis was applied to diagnose the noise sources and paths for the vehicle model described above, but with a different structural design of the chassis system and vehicle body. The energy participation factors of the component modes and substructures were calculated for simulated road inputs for which a low-frequency "boom" noise response was evident between 48-60 Hz. Figure 5 shows a plot of the participation factors of the vehicle body modes to the acoustic response at 54 Hz for this condition. The 49 Hz body mode predominates at this frequency and, in fact, is a major participant in generating the interior noise over the 48-60 Hz frequency band as indicated in Fig. 6. To obtain more detailed information on the noise sources in the vehicle, the energy participation factors of individual body panels were also calculated from Eq. (16). To illustrate, Fig. 7a shows a perspective view of boundary surface of the passenger compartment cavity and the corresponding distribution of the energy participation factors of the surrounding panels to the total noise response at 54 Hz. The darker areas in the figure represent those panels with greater participation factors. One observes from this plot that the front windshield and the right-rear roof are the major panels in producing the noise response. The participation of these panels, of course, resulted from the excitation of the 49 Hz body mode.

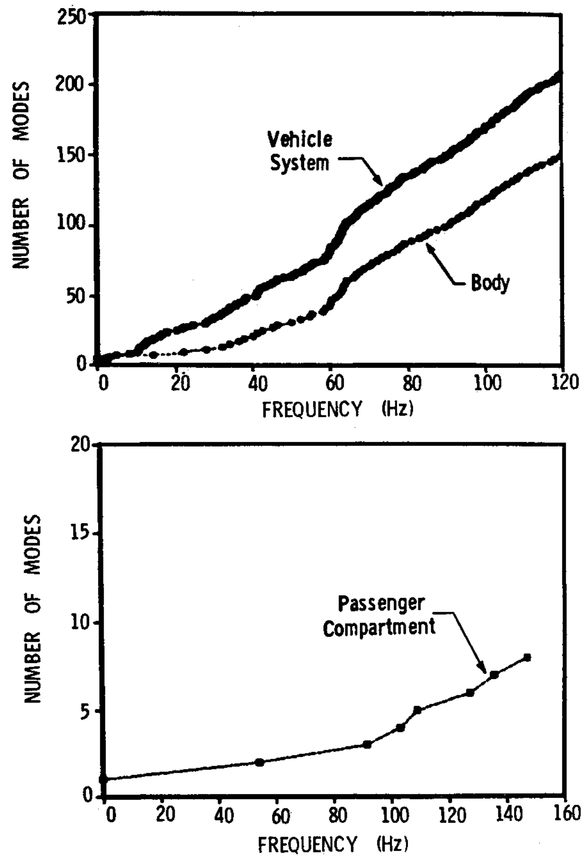


Fig. 3 Computed structural and acoustic modes.

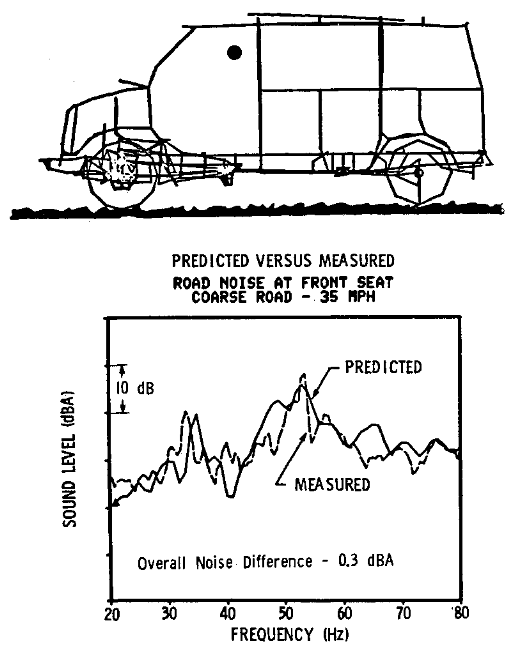


Fig. 4 Predicted vs measured noise in vehicle.

Based on this participation data, structural modifications were made to the vehicle body to reduce the vibration response of the problem panels. In addition, structural modifications were also made in the chassis system to reduce the energy transmitted to the body structure between 48-60 Hz. By incorporating these modifications in the vehicle, the vibration transmission path was altered and the total noise response was reduced. As shown in Fig. 7b, the participation factors for the modified vehicle indicate that the noise con-

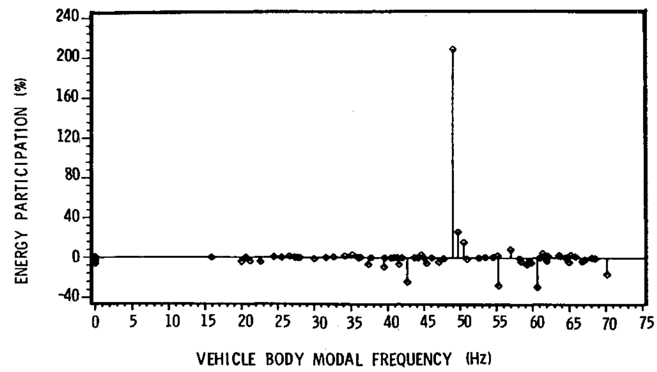


Fig. 5 Sound energy participation factors of vehicle body modes to acoustic response at 54 Hz.

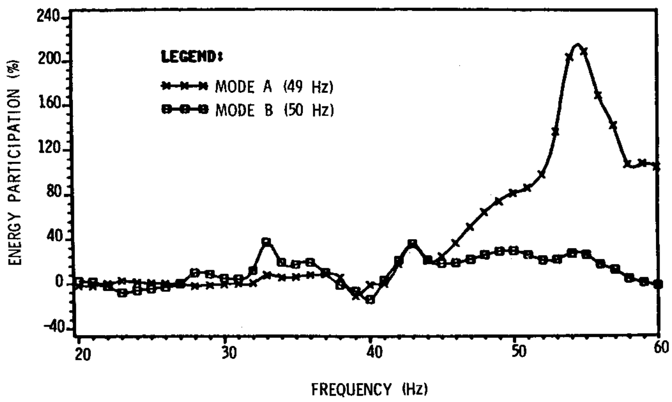


Fig. 6 Sound energy participation factors of two dominant vehicle body modes vs excitation frequency.

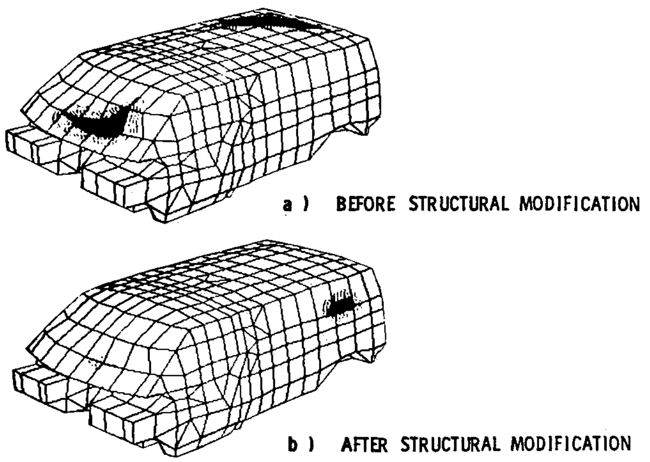


Fig. 7 Vehicle body panel participation to noise response at 54 Hz.

tribution due to front windshield and right-rear roof panel vibration were reduced significantly, as evidenced by the disappearance of the darker area. However, the figure also shows that the left-rear side panel now dominates the noise response in the modified vehicle. The overall effect of the structural modifications is illustrated in Fig. 8, which compares the interior noise response in the modified vehicle with that in the unmodified vehicle. The figure illustrates that the interior noise in the modified vehicle was actually reduced by over 10 dB at frequencies in the 48-60 Hz range as a result of the structural changes. In addition, the overall noise level in the vehicle was reduced by 7 dBA in the 20-60 Hz frequency range.

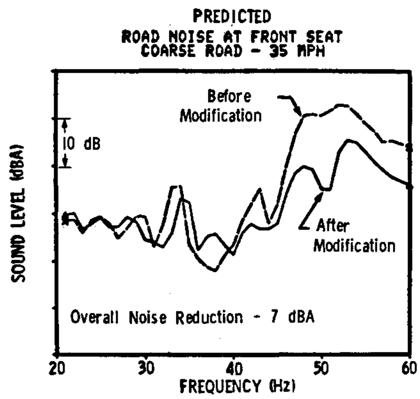


Fig. 8 Vehicle interior noise reduction due to structural modification.

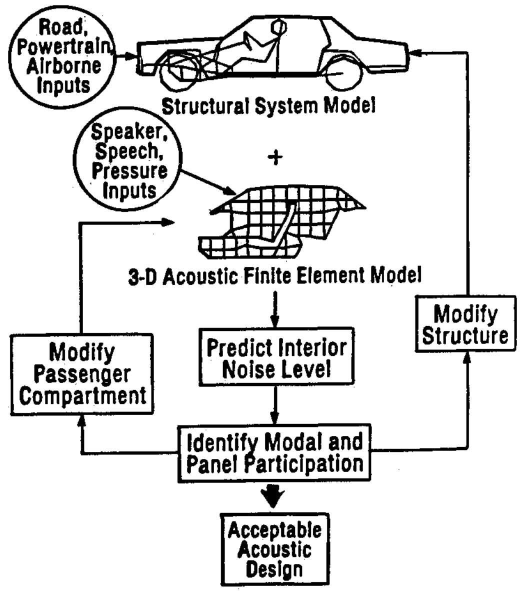


Fig. 9 Computer-aided structural-acoustic design procedure.

Structural-Acoustic Design Procedure

In practice, the structural-acoustic system model is applied in a systematic computer-aided procedure to minimize low-frequency interior noise in the automotive vehicle.[23,24] As summarized in the flow chart in Fig. 9, the vehicle structure and the passenger compartment cavity are modeled in the design stage using the finite element and modal methods. These two models are then combined mathematically to form a structural-acoustic system model that is used to predict the interior noise response for various input excitations existing under operating conditions. For the problem noise responses, the model can be used to identify the participation of the structural and acoustic modes or substructures consisting of the compartment wall panels. Based on these participation data, one may then determine whether the acoustic compartment or the vehicle structure should be modified. The acoustic compartment could be modified by applying or relocating absorption material or by changing the compartment boundary configuration. The vehicle structure could be modified with structural stiffeners, mass or damping treatment, or perhaps by tuning the various joint stiffnesses, mount properties, mount locations, etc.[25] The modifications are then evaluated using the system model and the procedure is repeated until the problem is resolved and an acceptable acoustic design is obtained. In this manner, one can arrive at the near-optimum structural and acoustic configuration for the passenger compartment in the design stage of the vehicle.

Conclusion

The component mode synthesis procedure was applied in this paper to develop a structural-acoustic system model of the automotive vehicle. The theoretical formulation to couple the modes of the passenger compartment cavity with the modes of the vehicle structural system was developed. In this formulation, the structural system modes were obtained by applying the component mode synthesis method to combine the vehicle body model and the chassis system model. Based on this methodology, an analytical structural-acoustic system model of the complete vehicle was synthesized to predict the low-frequency interior noise in the passenger compartment for realistic loading inputs to the vehicle. Finally, an analysis procedure was derived to calculate the energy participation level of component modes and vehicle body panels to the interior noise response.

To demonstrate these analytical capabilities, a structural-acoustic system model of a representative vehicle was developed and applied to predict the interior noise response for random road input. The results were compared with the measured noise in an actual vehicle and good agreement was observed for frequencies up to 80 Hz. By employing the model, the energy participation levels of the vehicle body modes and panels to the interior noise response were calculated for a particular noise problem. The model was applied to evaluate modifications that reduced the noise response and was demonstrated to be an effective design tool. Finally, the paper summarized the application of the model in a computer-aided procedure for the structural-acoustic design of the vehicle.

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